realization methods for the BLMS ADF's using the FNT and the FFT have been compared. Since the computation of the FNT is much faster than the computation of the FFT, one can conclude that the FNT realization of BLMS ADF is computationally more efficient than the FFT realization. Also, through the computer simulation of three practical applications, the convergence properties of the BLMS ADF's using the FNT and using the fixed-point FFT have been evaluated. The results of the simulation strongly indicate that the performance of the BLMS ADF's using the 16-bit FNT are comparable to those corresponding to the infinite precision FFT case, while the performances of the BLMS ADF's using the fixed-point FFT (such as 8-bit FFT) degrade fast as the transform length increases. Consequently, for the applications which require realization of an ADF with limited word length, the FNT provides an efficient realization method with good convergence properties.

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On Iterative Evaluation of Extrema of Integrals of Trigonometric Polynomials

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Abstract—**An iterative algorithm for evaluation of extrema of integrals of polynomials is presented. Each iteration requires two Fourier transforms instead of the matrix multiplication that is conventionally used. For rectangular windows, the results are eigenvalues of digital prolate functions. The corresponding eigenfunctions are also generated.**

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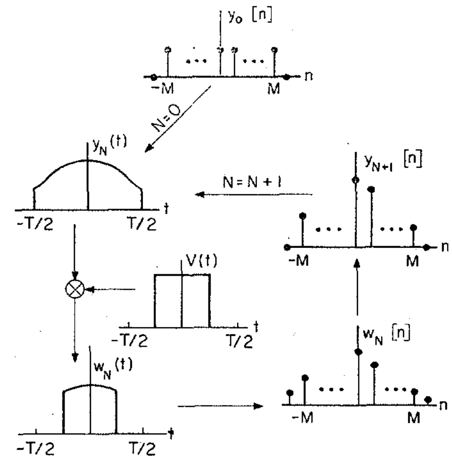

Fig. 1. Illustration of an iterative algorithm to generate maximum eigenvalue and corresponding eigenfunction.

Introduction

In this correspondence, we present an algorithm to either minimize or maximize

$$\alpha = \frac{1}{E} \int_{-T/2}^{T/2} v(t) |y(t)|^2 \, dt \tag{1}$$

where $v(t)$is a specified real function, $y(t)$ is a $(2M + 1)$st-order trigonometric polynomial

$$y(t) = \sum_{n=-M}^{M} y[n] \, e^{jn\omega_0 t}; \qquad \omega_0 = 2\pi/T$$

and

$$E = \int_{-T/2}^{T/2} |y(t)|^2 \, dt.$$

The maximum and minimum of (1) will be denoted, respectively, by $\overline{\alpha}$ and $\underline{\alpha}$.

Define $p_\tau(t)$ as unity for $|t| < \tau$ and zero elsewhere. For $v(t) = p_\tau(t)$ (with $\tau < T/2$), the extrema of (1) correspond to the extrema of eigenvalues of appropriately parameterized digital prolate functions [1], [2]. Applications include digital filter design [3]–[5] and spectral estimation [6]. The analog equivalent of the problem has been considered for rectangular [1, pp. 205–212], [7], [8], and triangular [9] shaped $v(t)$'s.

Papoulis [1] has shown that the extreme solutions of (1) are the extrema of the eigenvalues of the Toeplitz set of equations

$$\sum_{k=-M}^{M} v[n-k] \, y[k] = \lambda y[n]; \qquad |n| \le M \tag{2}$$

where $v[n]$ is the nth Fourier coefficient of $v(t)$ for $|t| \le T/2$. Thus, $\overline{\alpha} = \lambda_{\max}$ and $\underline{\alpha} = \lambda_{\min}$. Note that if we define $\hat{v}(t) = 1 - v(t)$, then $\hat{v}[n] = \delta[n] - v[n]$. Substituting into (2), we see that the maximum eigenvalue corresponding to $\hat{v}$ is the minimum eigenvalue corresponding to v. Hence, only an algorithm for finding maximum values is needed.

Fourier Algorithm

An iterative algorithm for finding $\overline{\alpha}$ and the corresponding $y(t)$ is shown in Fig. 1. Beginning with some initialization, we form the trigonometric polynomial

$$y_N(t) = \sum_{n=-M}^{M} y_N[n]\, e^{jn\omega_0 t}$$

where N parameterizes the iteration. This is multiplied by $v(t)$ to form the function

$$w_N(t) = y_N(t)v(t) = \sum_{n=-M}^{M} w_N[n]\, e^{jn\omega_0 t}$$

Multiplication of periodic functions is equivalent to convolving their Fourier coefficients. Hence,

$$w_N[n] = \sum_{k=-M}^{M} v[n-k]\, y_N[k]. \tag{3}$$

We keep only the $|n| \leq M$ terms and normalize to a unit norm:

$$y_{N+1}[n] = \begin{cases} w_N[n]/\sqrt{E_N}; & |n| \leq M \\ 0; & |n| > M \end{cases} \tag{4}$$

where

$$E_N = \sum_{n=-M}^{M} |w_N[n]|^2.$$

The cycle is again repeated. Under very loose conditions, $y_\infty(t)$ maximizes (1) with $\overline{\alpha} = \sqrt{E_\infty}$.

The proof is straightforward. Combining (3) and (4) gives

$$y_{N+1}[n] = \frac{1}{\sqrt{E_N}} \sum_{k=-M}^{M} v[n-k]\, y_N[k]; \qquad |n| \leq M. \tag{5}$$

The proof follows immediately from Von Mises' theorem [10].

If $|v(t)|\ 1 \leq 1$, the only step in Fig. 1 that adds energy is normalization by E_N. Without this step, the algorithm would converge to zero. We can, however, perform, say, P iterations without normalizing, and then normalize on the $P + 1$st iteration. The result would clearly be the same as if we normalized in each of the $P + 1$ iterations.

Convergence of the Fourier algorithm can be bettered further by employing techniques applicable to the accelerated Von Mises method, e.g., Wilkerson's method or relaxation parameters [11]. For a given problem, the Von Mises technique requires one matrix-matrix multiplication per iteration. The Fourier algorithm requires two more computationally efficient Fourier transforms (FFT's).

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Correction to "Long Convolutions Using Number Theoretic and Polynomial Transforms"

G. MARTINELLI

In the above paper,[1] the following corrections should be made in Fig. 1.

The block "decomposition by polynomial transforms of the polynomial . . . , the block "reduction of $H(z)X(z) \bmod (z^B + 1)$," and the block "exchange $2 \leftrightarrow z$ and transfomation of the polynomial products . . . " should be replaced by a unique block and should read as follows.

Decomposition by polynomial transforms of the polynomial products $\bmod(z^{2^h} + 1)$, $b + 2 \leq h \leq t - 1$, into $2^{t-b} - 4$ polynomial products $\bmod(z^{2^{b+1}} + 1)$, i.e.,

$$H_i(z)X_i(z) \bmod (z^{2^{b+1}} + 1), \qquad i = 1, 2, \cdots 2^{t-b} - 4.$$

The block "computation of the convolutions of length $B \bmod(F_b)$ by FNT" should read:

$$H_i(z)X_i(z) \qquad \bmod (z^{2^{b+2}} - 1)$$

computation of the convolutions by the FNT.

The block "exchange $z \leftrightarrow 2$" should read:

reductions mod $(z^{2^{b+1}} + 1)$.

The block "computation of the convolution of length B mod $(2^B - 1)$" should read:

computation of the convolution by the FNT

Finally, the quantity B must be replaced by 2^{b+2}.

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Passive Depth Tracking of Underwater Maneuvering Targets

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***Abstract*—As a parallel extension to the adaptive range tracking of underwater targets described by Moose and Dailey [1], this paper discusses the problem of tracking the depth of a maneuvering target using passive time-delay measurements. The target is free to maneuver in velocity and make random depth changes at times unknown to the ob-**

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